

THE COLLEGE OF AERONAUTICS.

C R A N F I E L D

The Diffusion of Loads in non-rigid

Circular Frames

-by-

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SUMMARY

This report extends the work of W.J. Goodey,⁽¹⁾ and gives numerical examples of the shear distribution around a frame subjected to a single concentrated radial load for variations in the parameters, such as, frame stiffness, skin thickness, stringer spacing, etc.

It also indicates when the beam theory distribution of shear can be used with a reasonable degree of accuracy.

The report contains a number of curves, figs. 5-17 giving the shear distribution around a frame for a single concentrated radial load of 1000 lb. The parameters chosen are those common to aircraft design, and it is possible to obtain a reasonably accurate shear distribution around a frame from the data supplied, without doing the actual shear calculation.

The case chosen is that of a long cylinder with a closed end, or that of a long cylinder where the portion aft of the loaded frame has a restraining effect upon the forward section, see fig.1.

The appendix gives the method of obtaining the shear load due to a tangential load and moment from the radial load expressions.

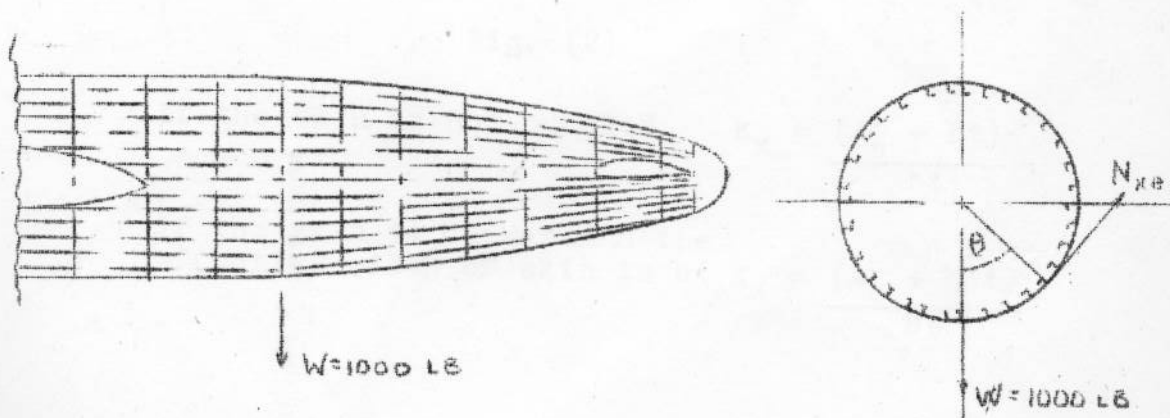


Fig. (1)

NOTATION

The notation used is:

- R = the radius of the frame at the skin line. ins.
 t = the actual skin thickness ins.
 I_f = the moment of inertia of the adjacent frames⁴ about an axis parallel to the skin line ins.⁴
 rI_f = the moment of inertia of the loaded frame about an axis parallel to the skin line ins.⁴
 L = frame spacing ins.
 ν = Poissons ratio, value taken as 0.3.
 $N_{x\theta}$ = shear per ins in the skin.
 $m = \frac{R}{L}$
 $e = 3 m^2 \{ 2 k_x (1 + \nu) - \nu \}$
 $\phi = \frac{R^3 k_x t}{I_f}$
 $c = 6 m^3 \phi$
 k_x = ratio of equivalent skin thickness to actual skin thickness, see fig. (2)

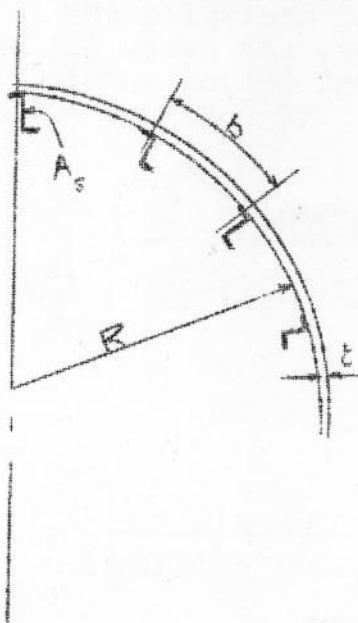


Fig. (2)

If the skin is not buckled $k_x = \frac{(A_s + bt)}{bt}$

If the skin is buckled and the effective width of skin is b' , $k_x = \frac{(A_s + b't)}{bt}$

INTRODUCTION

Goodey in his work considered two cases, namely (a) A long cylinder with the loaded frame at the free end.

(b) A long cylinder with the loaded frame at a distance from the free end.

The most practical case is that of (b) above, which was investigated by Goodey by considering a combination of the cases of a long cylinder loaded in the middle and a long cylinder loaded at its free end. See fig. (3).

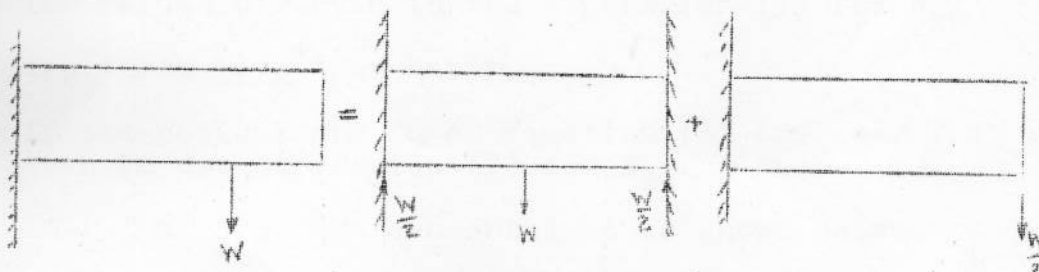


Fig. (3)

SOLUTIONS

The solutions contained in this report are for case (b) above the expression for the shear in the skin at the loaded frame, due to a radially applied load W is given by.

$$N_{x\theta} = \frac{W}{\pi R} \sum_{n=1}^{\infty} \left[\frac{\{(\lambda_2+1)(\lambda_1-1)^2 - (\lambda_1+1)(\lambda_2-1)^2\} n \sin n \theta}{(\lambda_2+1)(\lambda_1-1)^2 \left(\frac{r}{\lambda_1} + 2 - r\right) - (\lambda_1+1)(\lambda_2-1)^2 \left(\frac{r}{\lambda_2} + 2 - r\right)} \right] + \frac{W}{2\pi R} \sin \theta. \quad (1)$$

or

$$N_{x\theta} = \frac{W}{\pi R} \sum_{n=1}^{\infty} \left[\frac{a \{a^2 \sin 2\psi + a(a^2 - 2) \sin \psi - a \sin \psi\} n \sin n \theta}{\sin \psi \{2a^2(a^2 + 2a \cos \psi - 3) - r(a^4 - 6a^2 + 1) - 2ra(1 + a^2) \cos \psi\}} \right] + \frac{W}{2\pi R} \sin \theta. \quad (2)$$

The above expressions for $N_{x\theta}$ are given by Goodey, where for the loaded frame $k=1$.

In expression (1) above

λ_1 and λ_2 are obtained as shown below.

Solve the equation

$$X^2 + UX + V = 0 \quad (3)$$

where

$$U = \left\{ \frac{n^2(n^2-1)^2}{c} (n^2-2e) - 4 \right\} \quad (4)$$

$$V = 4 \left\{ 1 + \frac{n^2(n^2-1)^2}{c} (n^2+e) \right\} \text{-----}(5)$$

The roots of equation (3) are X_1 and X_2

Next solve the equations

$$\lambda + \frac{1}{\lambda} = X_1 \quad \text{and} \quad \lambda + \frac{1}{\lambda} = X_2 \text{-----}(6)$$

The roots of equations (6) are λ_1 and λ_3 , and λ_2 and λ_4 respectively, where $\lambda_1 = \frac{1}{\lambda_3}$ and $\lambda_2 = \frac{1}{\lambda_4}$.

The values of λ used in the expression (1) for N_{x0} are those less than unity.

If the roots X_1 and X_2 of equation (3) are less than 2.0 then we use expression (2) for N_{x0} .

"a" and " ψ " are obtained as shown below.

$$Y^2 - (V+4)Y + U^2 = 0 \text{-----}(7)$$

where U and V are given by expressions 4 and 5. The roots of equation (7) are Y_1 and Y_2 where one root is greater than 4.0 and one less than 4.0.

Let Y_1 be the root greater than 4.0 and Y_2 the root less than 4.0, we then solve the equations

$$(a + \frac{1}{a}) = +\sqrt{Y_1} \quad \text{and} \quad (2 \cos \psi)^2 = Y_2 \text{---}(8)$$

The values of "a" found are a_1 and a_2 where $a_1 = \frac{1}{a_2}$

and the value of "a" used in the expression (2) for N_{x0} is that which is less than unity.

The value of " ψ " used in the expression (2) for N_{x0} is the value given by Y_2 which is of opposite sign to "U" in expression (4). This is so, because

$$-U = 2(a + \frac{1}{a}) \cos \psi$$

the term $(a + \frac{1}{a})$ is always +ve hence $\cos \psi$ must be -ve, i.e. the sign of " ψ " must be opposite to that of "U"

The expression for N_{x0} can be written in the form

$$N_{x0} = \frac{W}{\pi R} \sum_{n=1}^{\infty} F(\lambda_1 \lambda_2 k, r) \quad n \sin n\theta + \frac{W}{2\pi R} \sin \theta \text{-----}(9)$$

or

$$N_{x0} = \frac{W}{\pi R} \sum_{n=1}^{\infty} F(a, \psi, k, r) \quad n \sin n\theta + \frac{W}{2\pi R} \sin \theta \text{-----}(10)$$

In both expressions 9 and 10 the value of $F(\text{----})$ $n \sin n\theta$ for $n=1$ is equal to $\frac{\sin \theta}{2}$

For practical purposes it is sufficiently accurate to take the $\sum_{n=1}$ term up to a value of $n=6$, hence the expression for shear can be written as

$$N_{x\theta} = \frac{W}{\pi R} \sin\theta + \frac{W}{\pi R} \sum_{n=2}^6 F(\text{----})_n \sin n\theta \text{ ----- (11)}$$

The curves of figs. 5-20 were obtained by putting numerical values in the expression (11) above. The parameters for each particular case are given in the accompanying curves.

I am indebted to Dr. Kirkby and the computing section of the Aerodynamics Department for the valuable aid given in the computation of the numerical examples.

Conclusions

- 1) The distribution of shear around the frame is not sensitive to variations in values of " k_x " the results show a maximum increase of 7% in value of $N_{x\theta}$ at 30° (i.e. maximum value) for a 100% increase in value of k_x , see figs. 15, 16 and 17.
- 2) The distribution of shear around the frame is not sensitive to variations in skin thickness " t ". In aircraft structures going from one gauge to the next is approximately an increase of 30% and for this increase in skin thickness the average percentage decrease in maximum value of $N_{x\theta}$ is 4%. This is assuming that k_x remains constant.
- 3) The distribution of shear is not sensitive for reasonably large variations in the moment of inertia of the adjacent frames, and to take a mean value of I_f is sufficiently accurate for the results. The effect can be obtained from fig. (5) where it is seen that increasing the stiffness of the adjacent frames, the loaded frame remaining constant increases the maximum value of $N_{x\theta}$. For the parameters chosen it is seen that the average percentage increase in value of $N_{x\theta}$ at 30° is 0.12% for a 1.0% increase in value of I_f .
- 4) The distribution of shear is not sensitive for reasonably large variations in the moment of inertia of the loaded frame, and a mean value is sufficiently accurate for the results. This effect can be obtained from figs. 12, 13, 14, 18, 19 and 20.
- 5) The distribution of shear is not sensitive for moderate variations in frame spacing. For a frame radius up to 40" the average increase in value of $N_{x\theta}$ at 30° is 0.4% for a 1.0% increase in value of " m " and for a radius of 60" the increase in value of $N_{x\theta}$ at 30° is 0.22% for a 1.0% increase in " m ". This effect can be obtained from figs. 18, 19, and 20.
- 6) The beam theory distribution of shear is sufficiently accurate if the value of rI_f lies above the line of fig. 21. This curve is obtained by extrapolation of the curves figs. 12, 13 and 14, and is intended to give the lowest value of rI_f for which the beam theory is a reasonable approximation.

7) The shear loading on frames adjacent to the loaded frame can be obtained with sufficient accuracy for practical purposes by interpolation. The shear distribution on a frame 3 frame spacings distant from the loaded frame can be taken as that given by the beam theory and intermediate frames can be obtained by a straight line interpolation between the frames, see fig. 4.

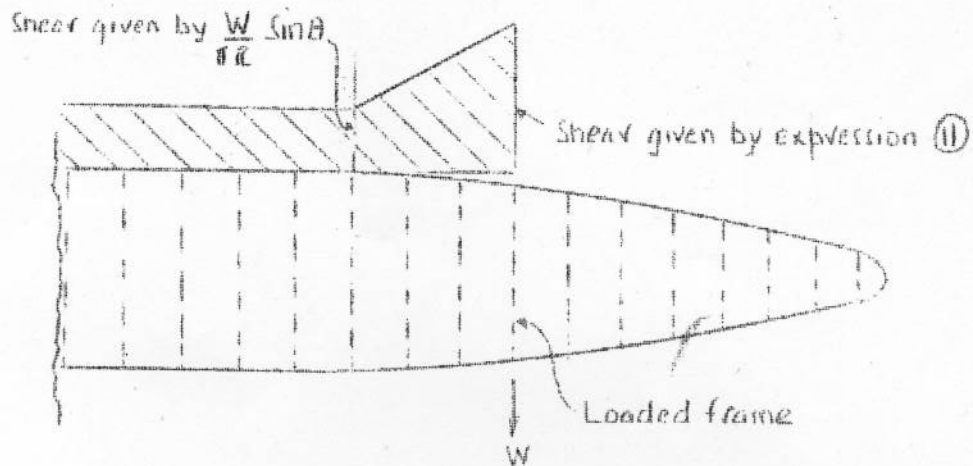


Fig. (4)

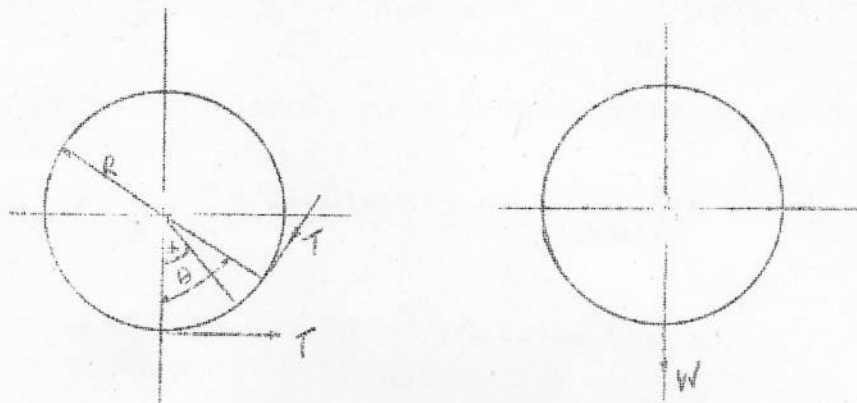
Reference

- (1) Royal Aeronautical Society Journal Nov. 1946.
"The stresses in a circular fuselage" by W.J. Goodey.

APPENDIX A

Derivation of the shear load due to tangential load and moment from the radial load expression.

Goodey in his work indicates a method by which the shear load for a radial load and moment may be obtained from the expression for a tangentially applied load. Here the complete solution is given for obtaining the shear load due to a tangential load and moment from the radial load expressions.



Let $(N_{x\theta})_T$ = shear load due to a tangentially applied load T

This will be a function of T and θ .

$$(N_{x\theta})_T = -Tf(\theta - \alpha) = -Tf(\theta) + Ta f'(\theta)$$

when α is small

$$Ta = W \quad \text{and} \quad T = T \cos \alpha.$$

$$\therefore \Delta(N_{x\theta})_T = Ta f'(\theta) = W f'(\theta) = (N_{x\theta})_W$$

$$\therefore (N_{x\theta})_W = \frac{d}{d\theta} \frac{W}{T} (N_{x\theta})_T$$

$$(N_{x\theta})_T = \left[(N_{x\theta})_T \right]_{\theta=0} + \frac{T}{W} \int_0^\theta (N_{x\theta})_W d\theta \quad \text{-----(12)}$$

where the term $\left[(N_{x\theta})_T \right]_{\theta=0}$ is a constant of integration

and can be evaluated by considering the equilibrium of the frame.

$$\begin{aligned} TR &= \int_0^{2\pi} (N_{x\theta})_T R^2 d\theta. \\ &= \left\{ (N_{x\theta})_T \right\}_{\theta=0} 2\pi R^2 + \frac{T}{W} R^2 \int_0^{2\pi} \int_0^\theta (N_{x\theta})_W(\phi) d\phi d\theta. \\ &= 2\pi R^2 \left\{ (N_{x\theta})_T \right\}_{\theta=0} + \frac{T}{W} R^2 \int_0^{2\pi} (2\pi - \phi) (N_{x\theta})_W(\phi) d\phi d\theta \end{aligned}$$

$$= 2\pi R^2 \left\{ (N_{x\theta})_r \right\}_{\theta=0} - \frac{T}{W} R^2 \int_0^{2\pi} \phi (N_{x\theta})_w \phi d\phi$$

$$\therefore \left\{ (N_{x\theta}) \right\}_{\theta=0} = \frac{T}{2\pi R} + \frac{T}{2\pi W} \int_0^{2\pi} \phi (N_{x\theta})_w (\phi) d\phi \text{ which gives}$$

the constant of integration for (12) above.

$$\therefore (N_{x\theta})_r = \frac{T}{2\pi R} + \frac{T}{2\pi W} \int_0^{2\pi} \phi (N_{x\theta})_w \phi d\phi + \frac{T}{W} \int_0^\theta (N_{x\theta})_w d\theta \quad \text{--- (13)}$$

For the case considered, of a loaded frame along a cylinder

$$(N_{x\theta})_w = \frac{W}{\pi R} \sum_{n=1}^{\infty} F(\text{----}) n \sin n\theta + \frac{W}{2\pi R} \sin \theta$$

$$= \frac{W}{\pi R} \sin \theta + \frac{W}{\pi R} \sum_{n=2}^6 F(\text{----}) n \sin n\theta$$

$$\therefore \frac{T}{2\pi W} \int_0^{2\pi} \phi (N_{x\theta})_w \phi d\phi = \frac{T}{2\pi W} \int_0^{2\pi} \theta \left\{ \frac{W}{\pi R} \sin \theta + \frac{W}{\pi R} \sum_{n=2}^6 F(\text{----}) n \sin n\theta \right\} d\theta$$

$$= \frac{T}{2\pi} \int_0^{2\pi} \theta \sin \theta d\theta + \frac{T}{2\pi W} \frac{W}{\pi R} \sum_{n=2}^6 F(\text{----}) \int_0^{2\pi} \theta n \sin n\theta d\theta$$

$$= \frac{T}{2\pi} \int_0^{2\pi} [\sin \theta - \theta \cos \theta]_0^{2\pi} + \frac{T}{2\pi} \sum_{n=2}^6 F(\text{----}) n \left[\frac{\sin n\theta - n\theta \cos n\theta}{n} \right]_0^{2\pi}$$

$$= \frac{T}{2\pi} \int_0^{2\pi} [-2\pi] + \frac{T}{2\pi} \sum_{n=2}^6 F(\text{----}) \frac{n}{n^2} [-2\pi n]$$

$$= -\frac{T}{\pi R} - \frac{T}{\pi R} \sum_{n=2}^6 F(\text{----}).$$

substituting in equation (13)

$$(N_{x\theta})_r = \frac{T}{2\pi R} - \frac{T}{\pi R} - \frac{T}{\pi R} \sum_{n=2}^6 F(\text{----}) + \frac{T}{W} \int_0^\theta (N_{x\theta})_w d\theta$$

$$= -\frac{T}{2\pi R} - \frac{T}{\pi R} \sum_{n=2}^6 F(\text{----}) + \frac{T}{W} \int_0^\theta \frac{W}{\pi R} \sin \theta d\theta$$

$$+ \frac{W}{\pi R} \sum_{n=2}^6 F(\text{----}) \int_0^\theta n \sin n\theta d\theta \}$$

$$\begin{aligned}
 &= -\frac{T}{2\pi R} - \frac{T}{\pi R} \sum_{n=2}^6 F(\text{-----}) - \frac{T}{W} \frac{W}{\pi R} \cos \theta + \frac{T}{W} \frac{W}{\pi R} \sum_{n=2}^6 F(\text{-----}) n \frac{1}{n} [\cos n\theta - 1] \\
 &= -\frac{T}{2\pi R} - \frac{T}{\pi R} \sum_{n=2}^6 F(\text{-----}) - \frac{T}{\pi R} \cos \theta \\
 &\quad + \frac{T}{\pi R} + \frac{T}{\pi R} \sum_{n=2}^6 F(\text{-----}) - \frac{T}{\pi R} \sum_{n=2}^6 F(\text{-----}) \cos n\theta \\
 &= \frac{T}{2\pi R} - \frac{T}{\pi R} \cos \theta - \frac{T}{\pi R} \sum_{n=2}^6 F(\text{-----}) \cos n\theta. \\
 &= \frac{T}{\pi R} \left[\frac{1}{2} - \cos \theta - \sum_{n=2}^6 F(\text{-----}) \cos n\theta \right] \text{-----} (14)
 \end{aligned}$$

—————0—————

Let $(N_{x\theta})_{\tau}$ = shear load due to an applied moment.

$$(N_{x\theta})_{\tau} = \frac{M}{R} \left\{ f(\theta)_{\tau} + \frac{d^2 f(\theta)_{\tau}}{d\theta^2} \right\}$$

where $f(\theta)_{\tau}$ is the shear due to unit tangential load.

$$f(\theta)_{\tau} = \frac{1}{\pi R} \left\{ \frac{1}{2} - \cos \theta - \sum_{n=2}^6 F(\text{-----}) \cos n\theta \right\} \text{ see (14) above}$$

$$f'(\theta)_{\tau} = \frac{1}{\pi R} \left\{ \sin \theta + \sum_{n=2}^6 F(\text{-----}) n \sin n\theta \right\}$$

$$\frac{d^2 f(\theta)_{\tau}}{d\theta^2} = \frac{1}{\pi R} \left\{ \cos \theta + \sum_{n=2}^6 F(\text{-----}) n^2 \cos n\theta \right\}$$

$$(N_{x\theta})_{\tau} = \frac{M}{R} \left\{ \frac{1}{2\pi R} - \frac{\cos \theta}{\pi R} - \frac{1}{\pi R} \sum_{n=2}^6 F(\text{-----}) \cos n\theta \right.$$

$$\left. + \frac{\cos \theta}{\pi R} + \frac{1}{\pi R} \sum_{n=2}^6 F(\text{-----}) n^2 \cos n\theta \right\}$$

$$= \frac{M}{\pi R^2} \left\{ \frac{1}{2} - \sum_{n=2}^6 (1-n^2) F(\text{-----}) \cos n\theta \right\}$$

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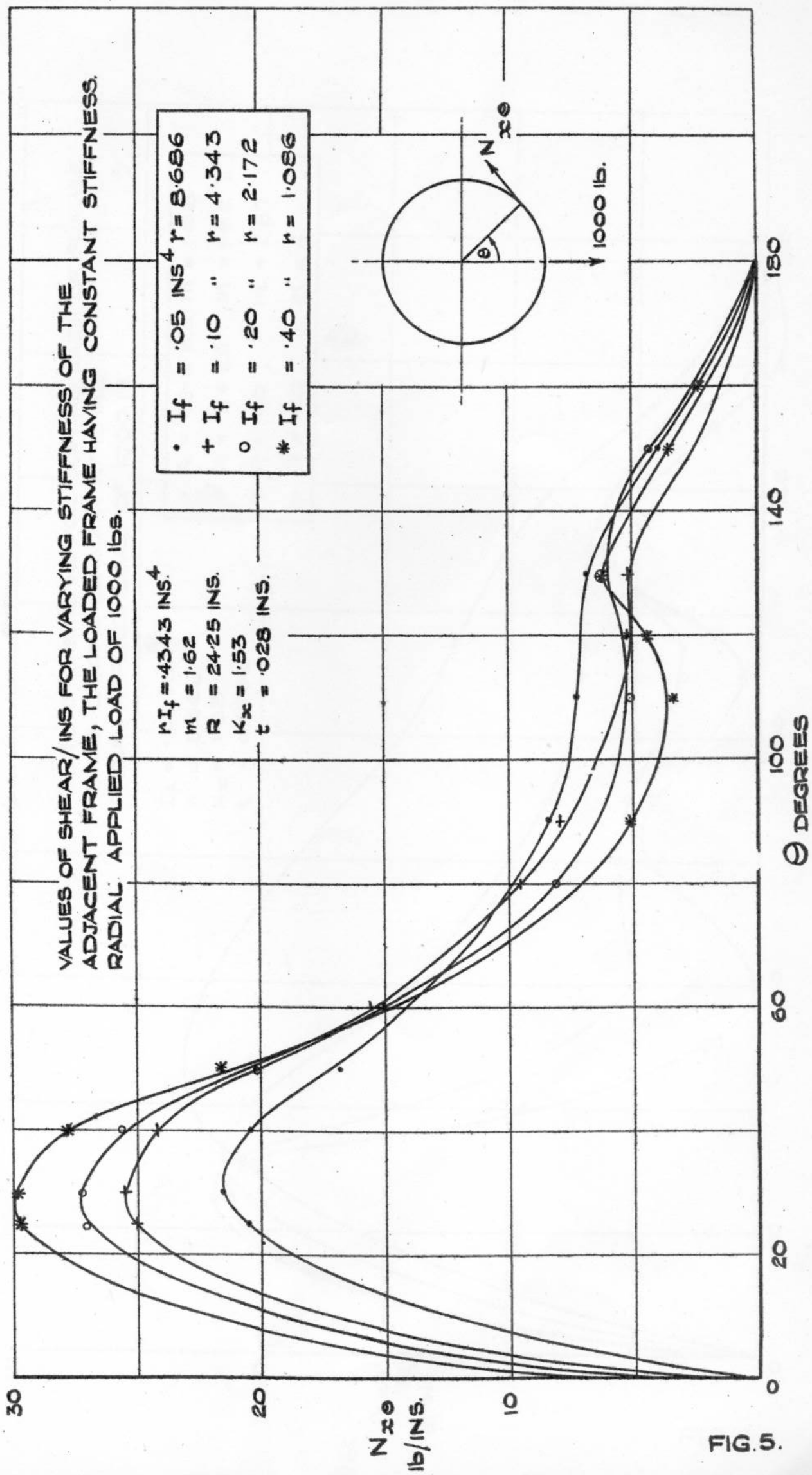


FIG. 5.

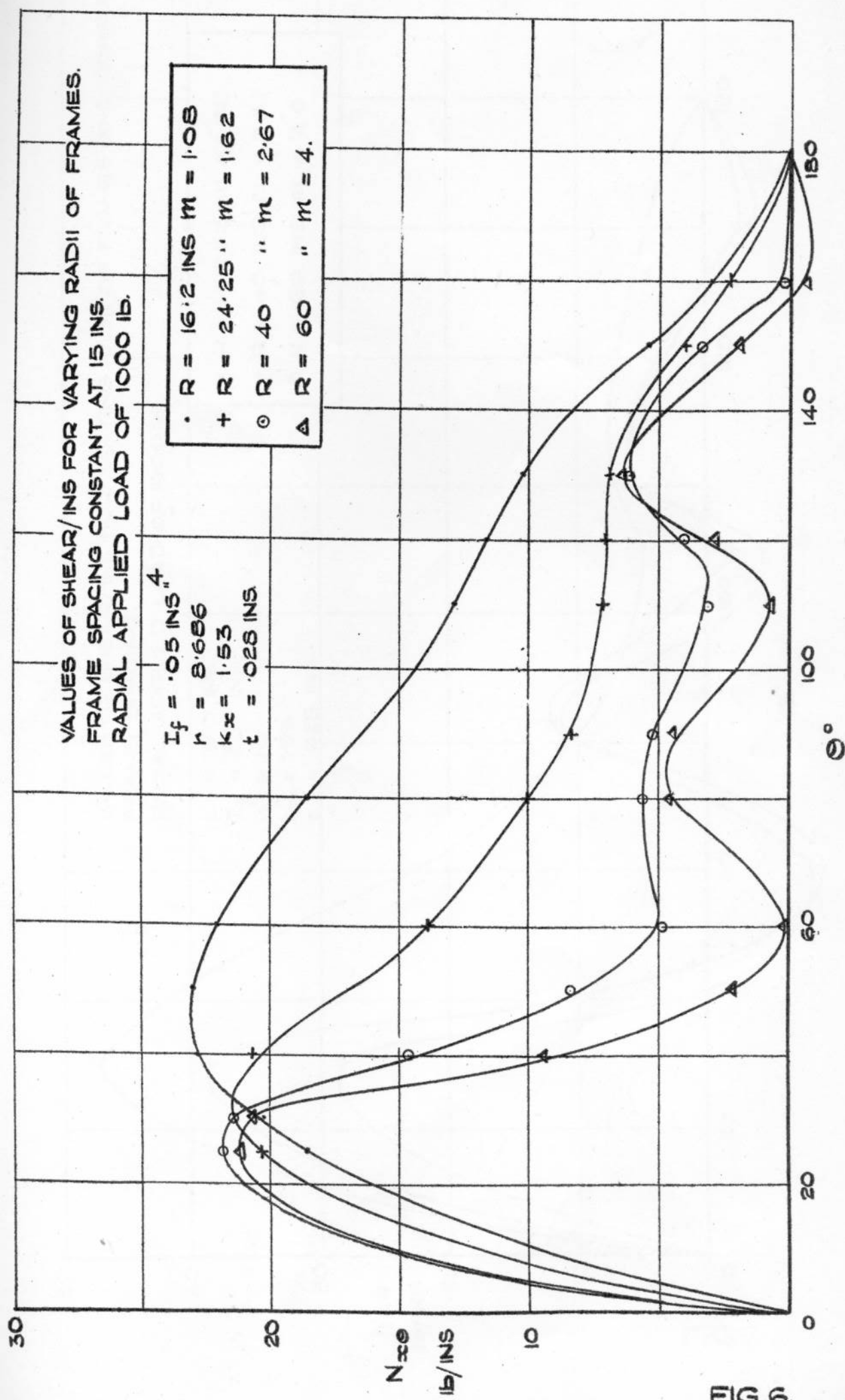


FIG.6.

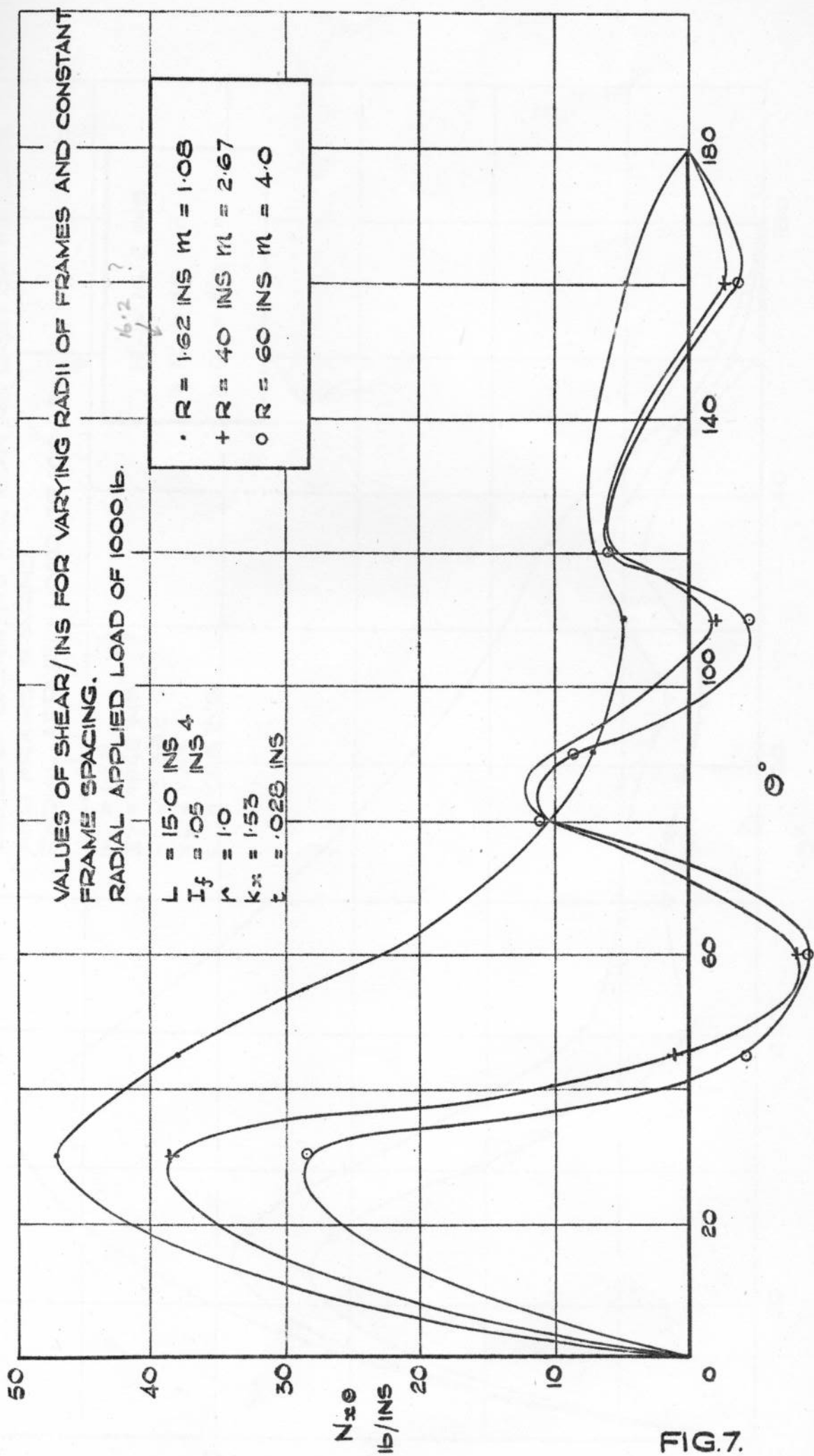


FIG.7.

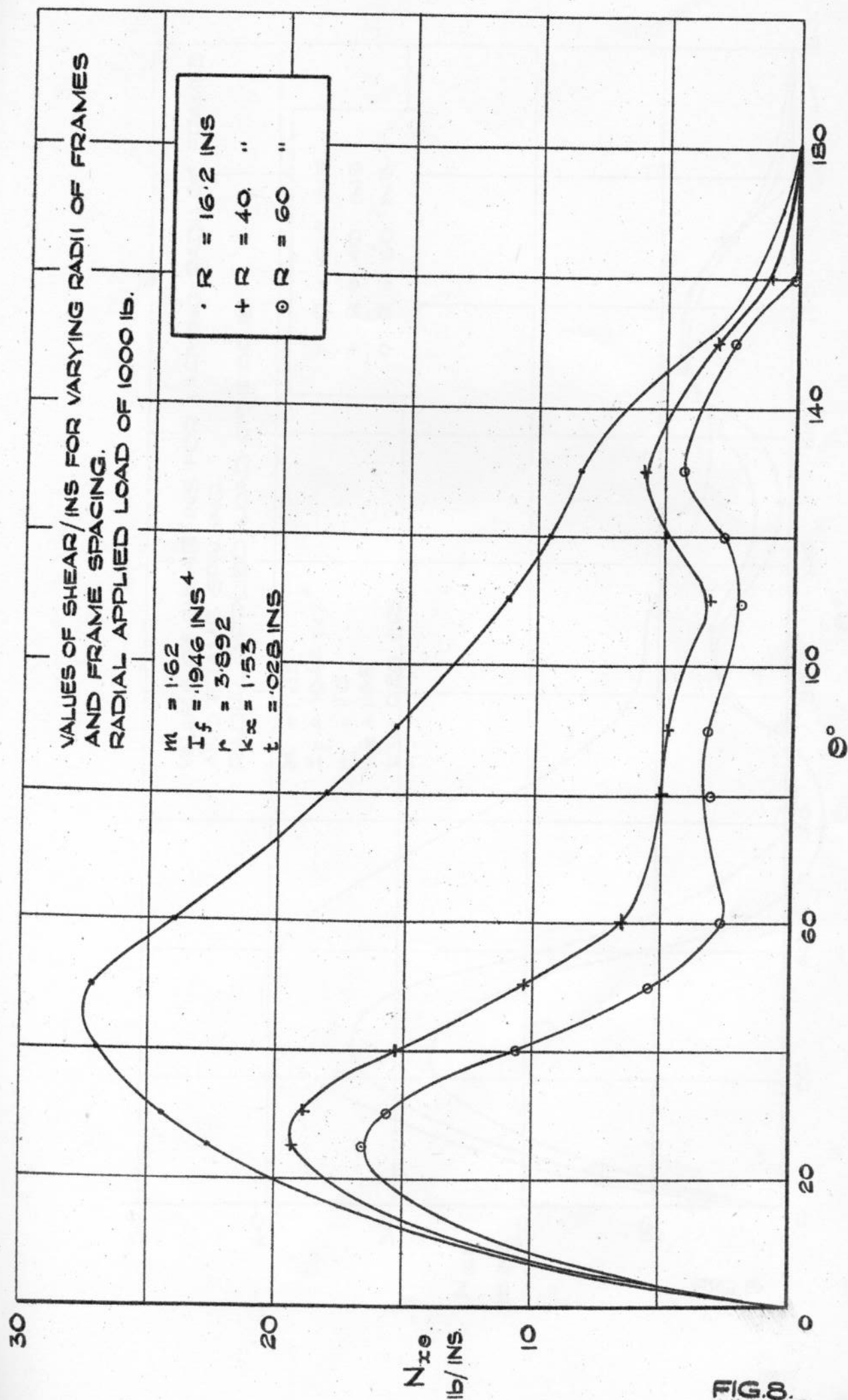


FIG. 8.

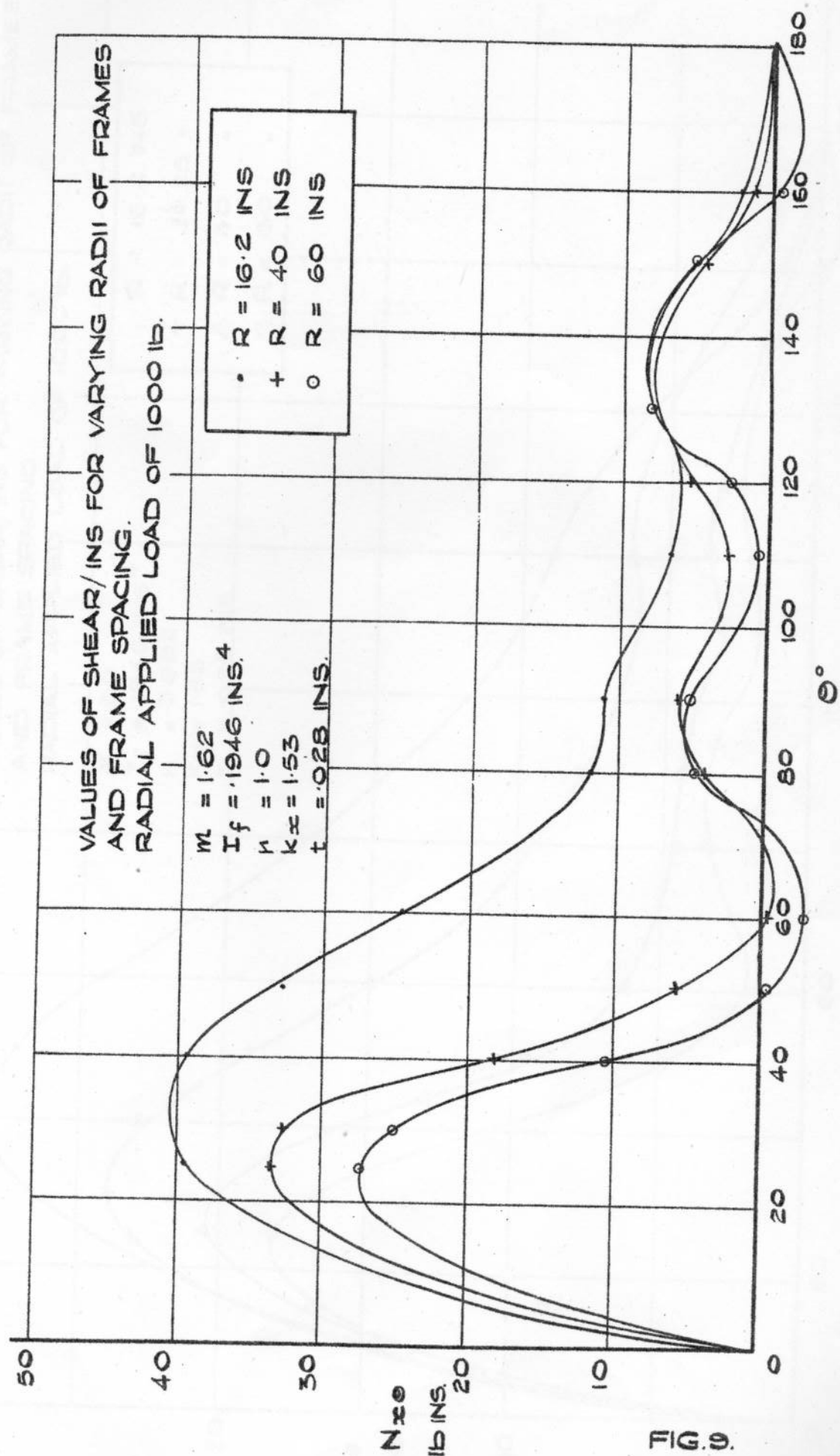


FIG. 9.

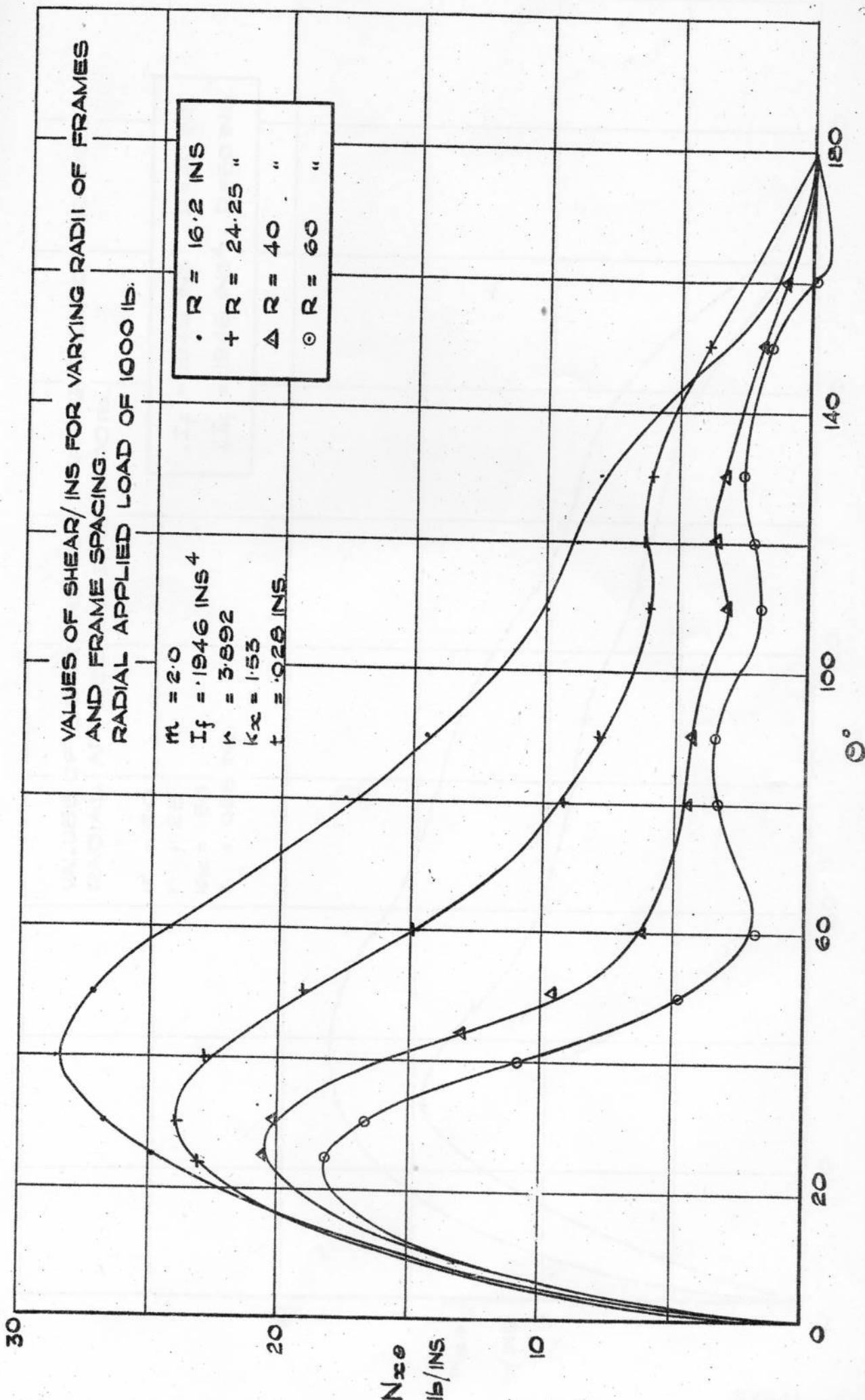


FIG.10.

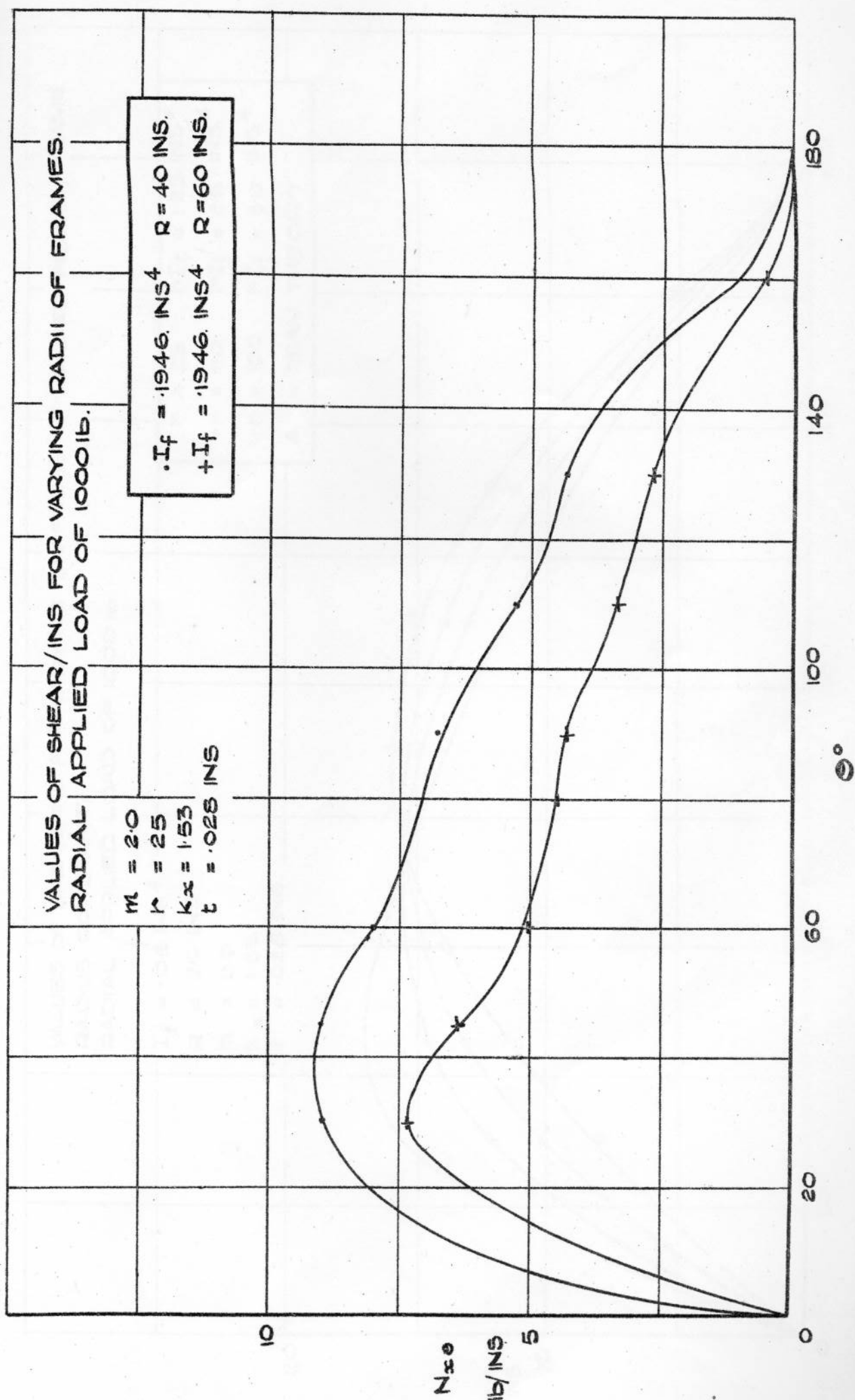


FIG. 11.

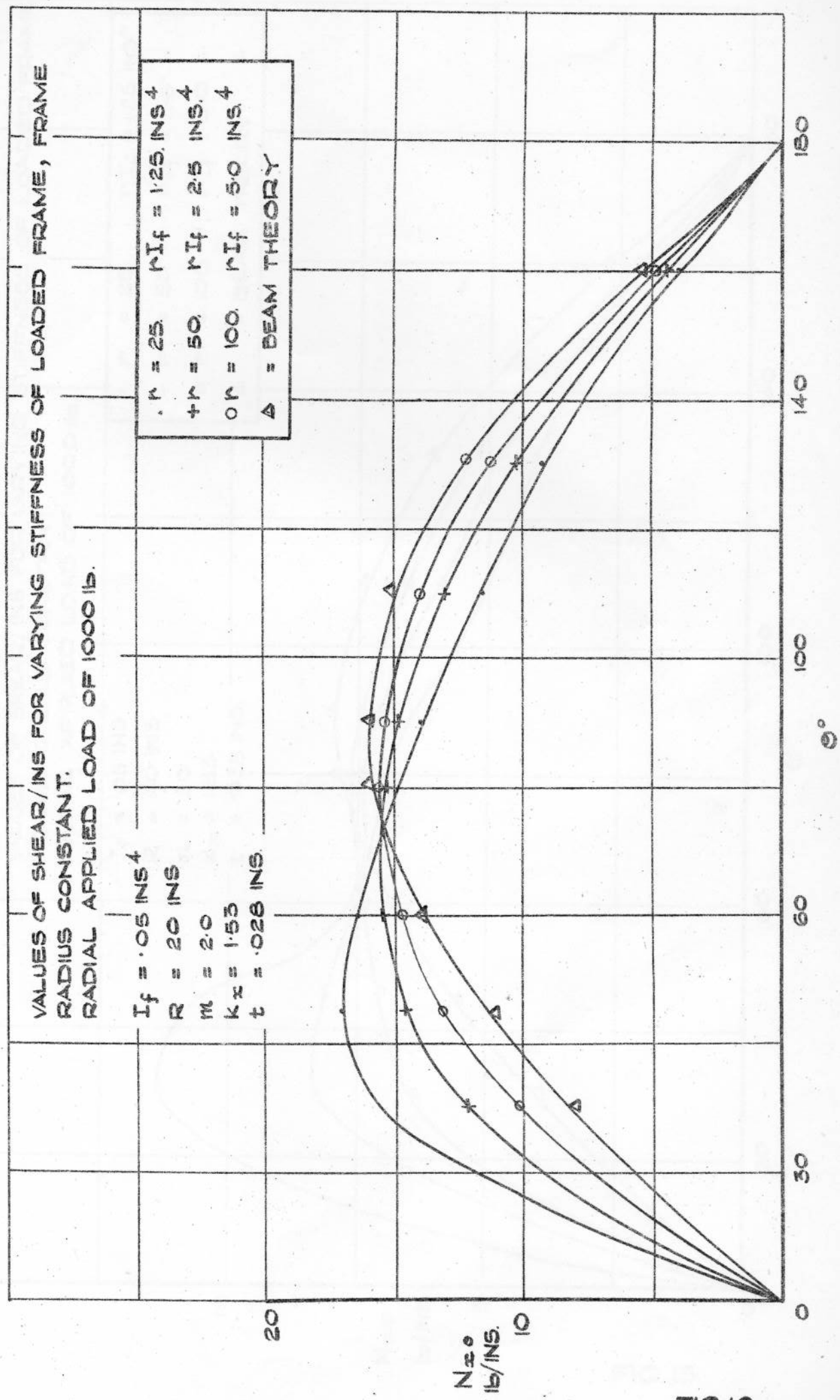


FIG.12.

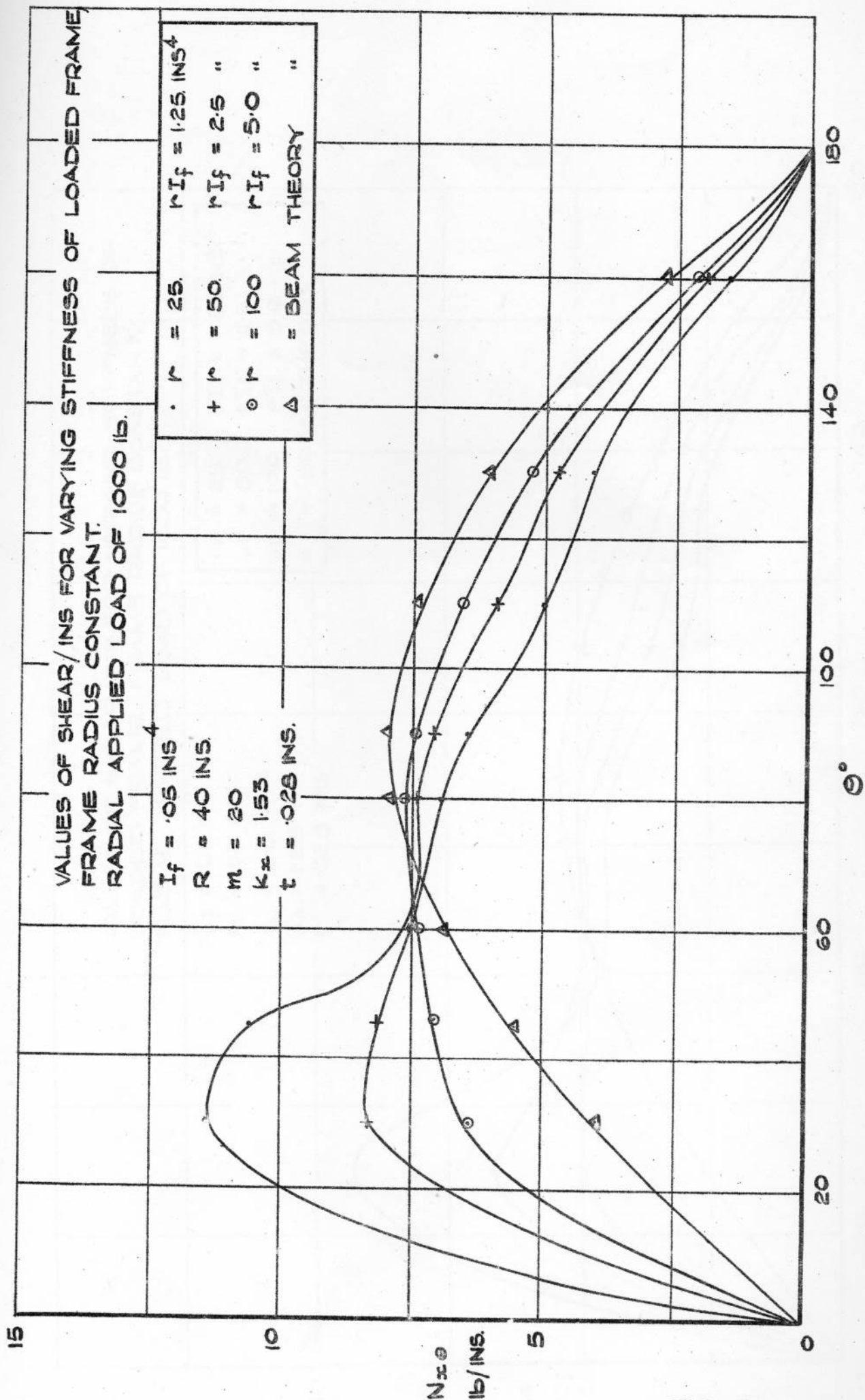


FIG. 13.

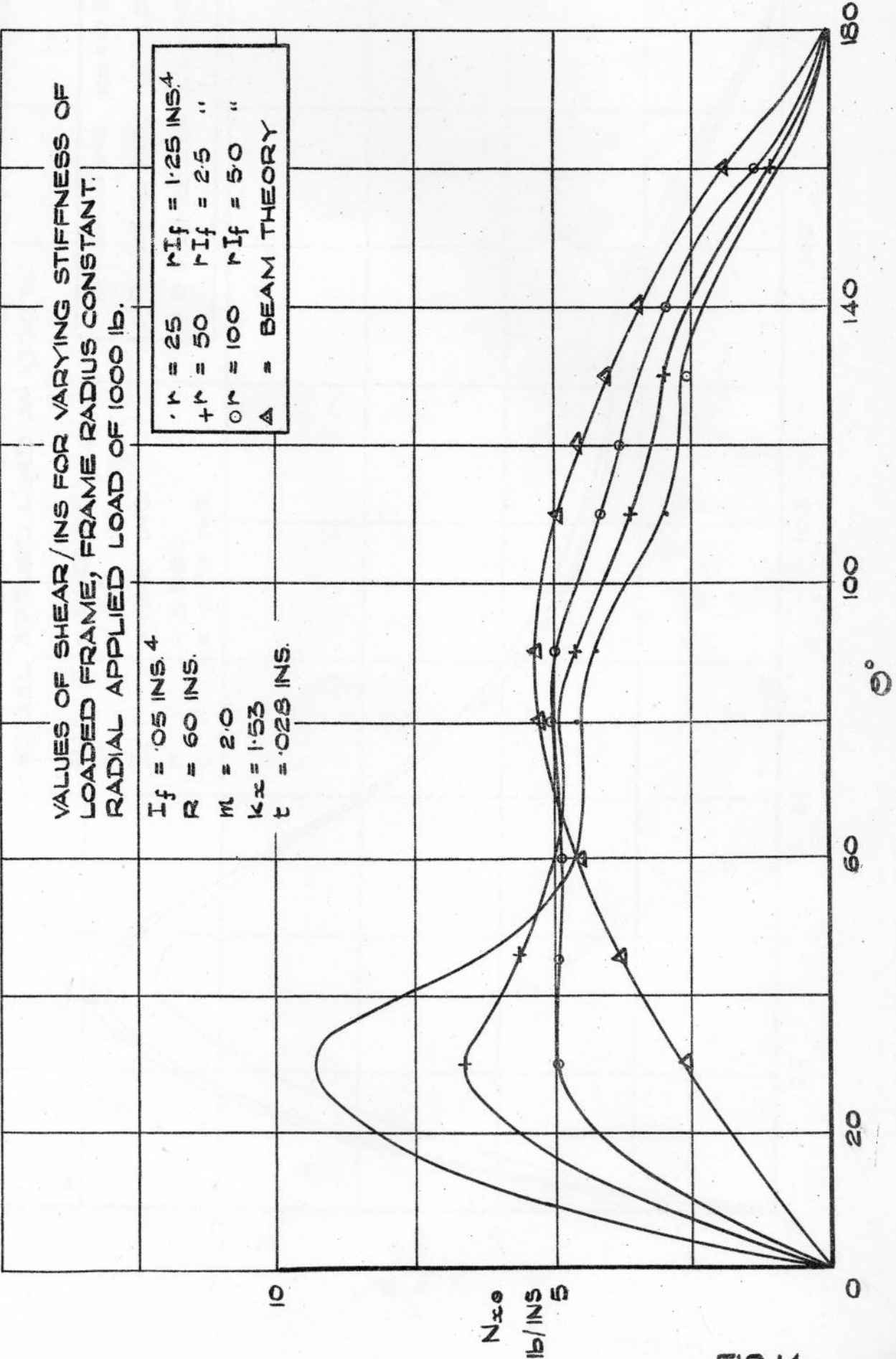


FIG.14.

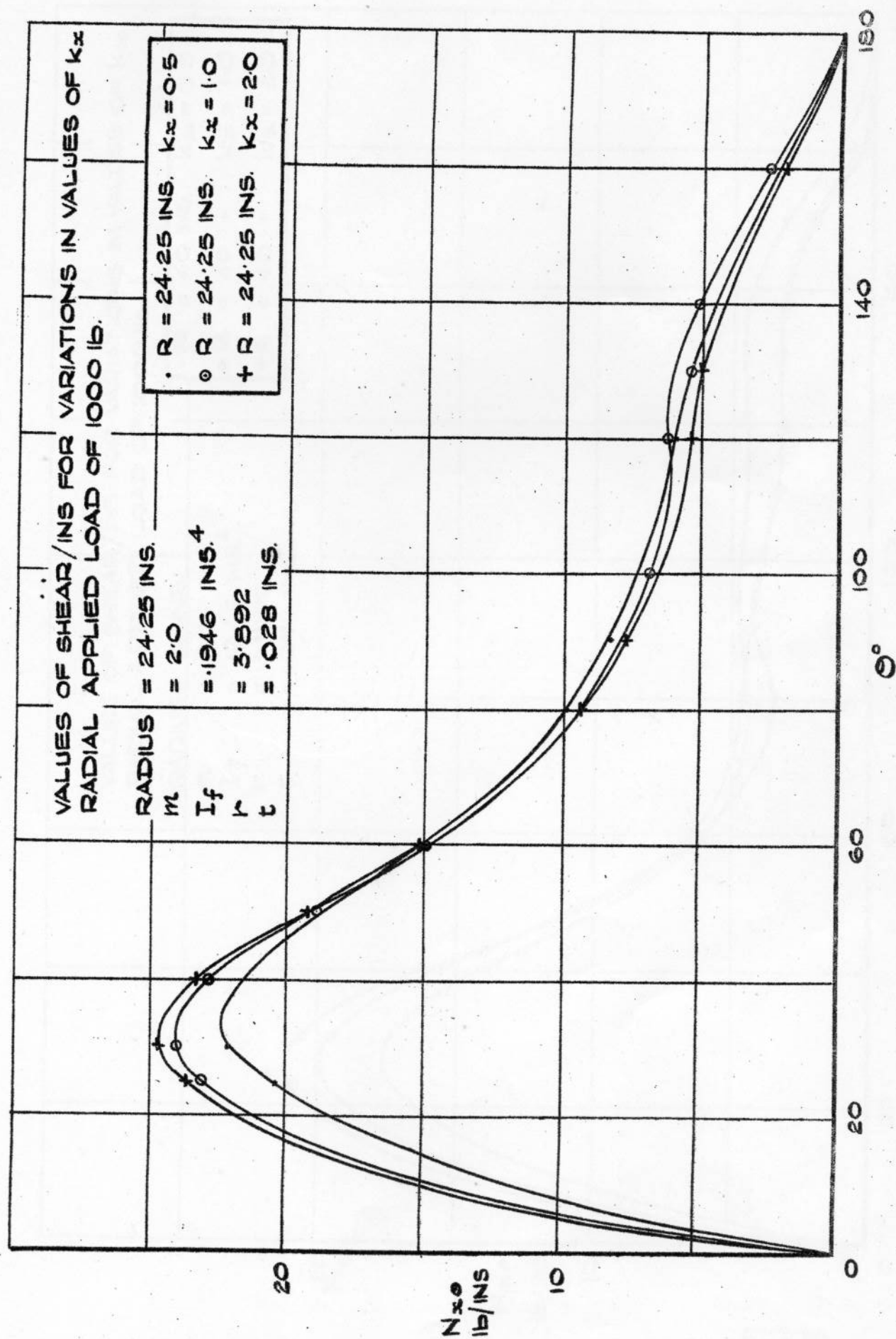


FIG. 15.

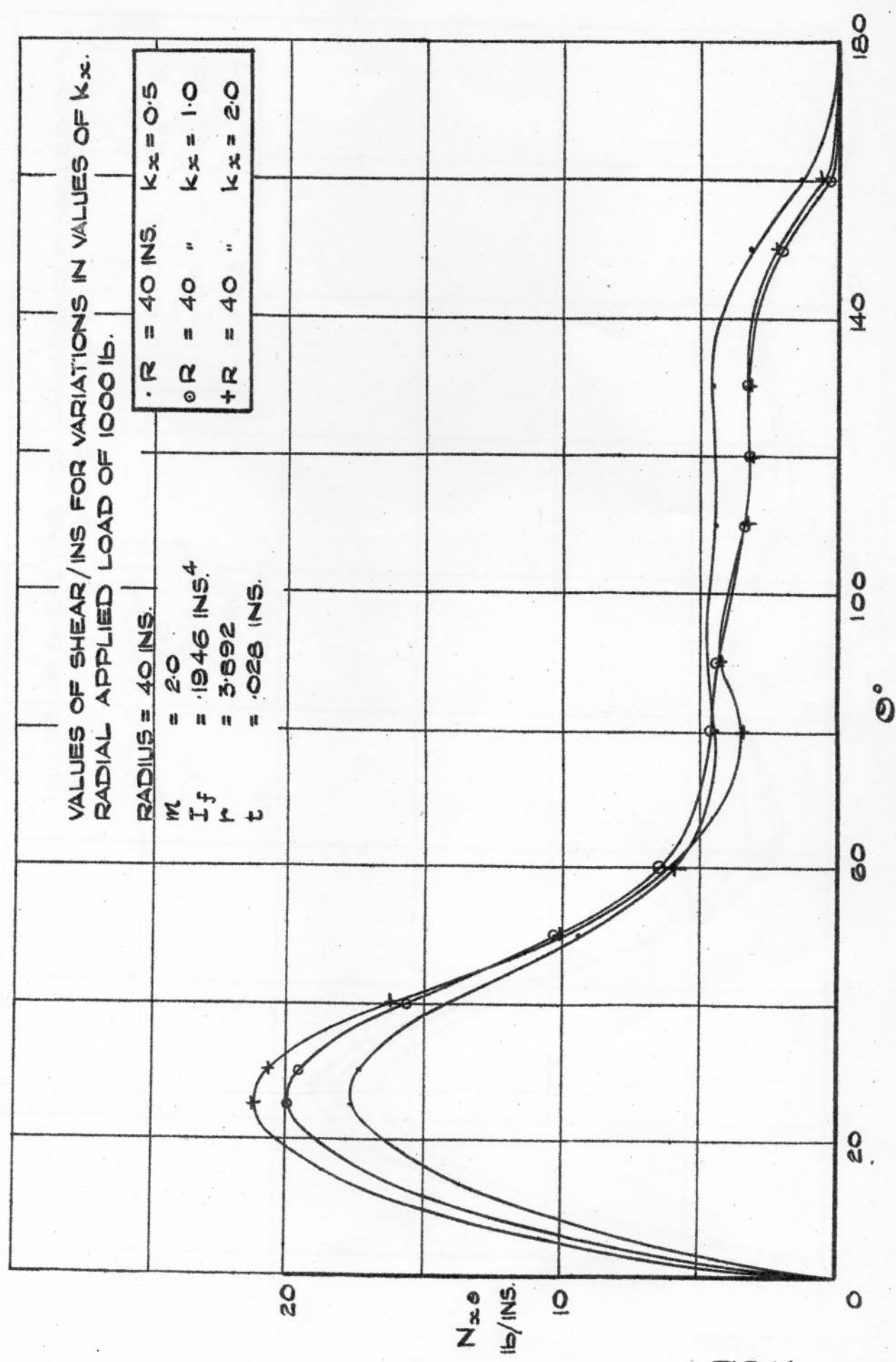


FIG.16.

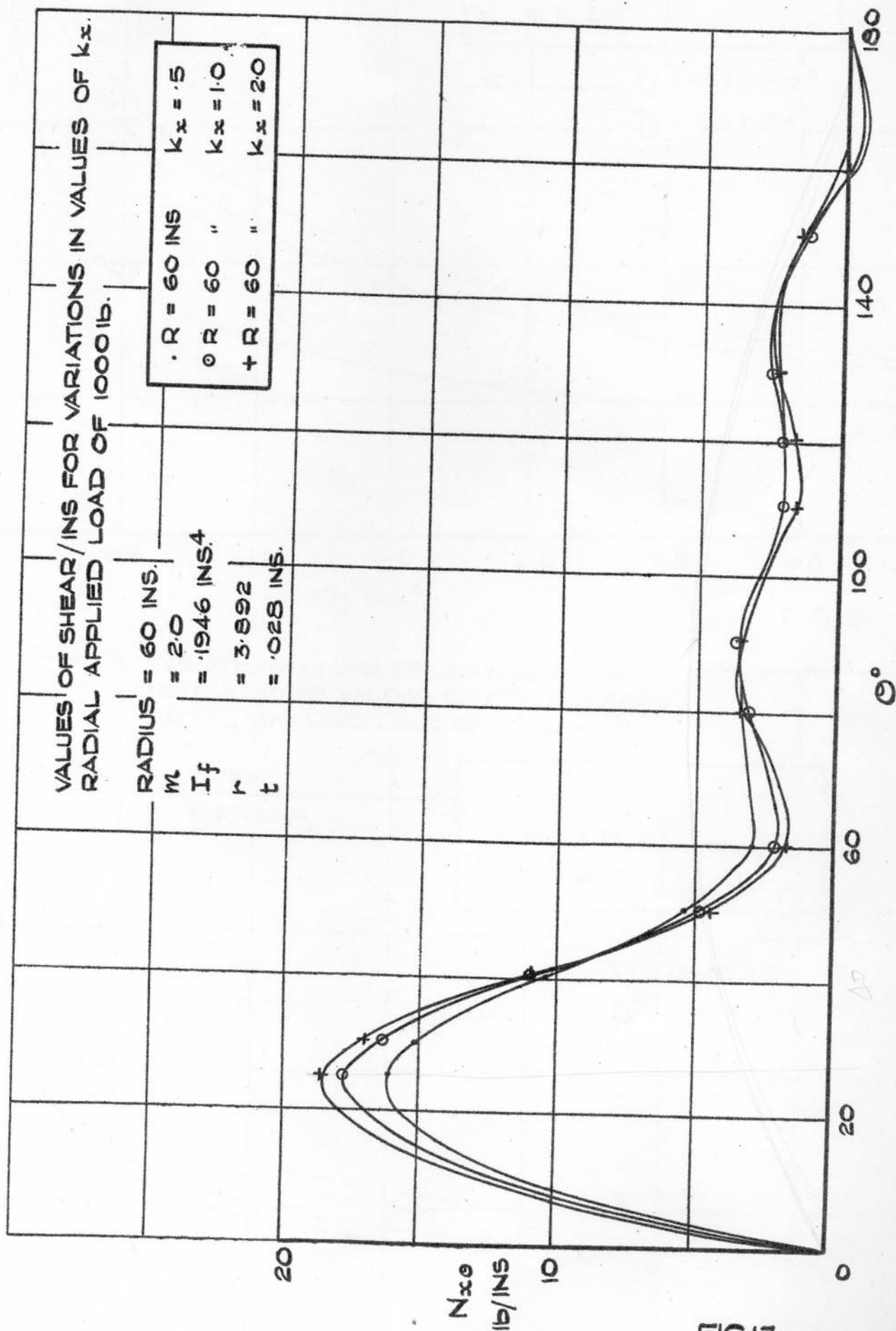


FIG.17.

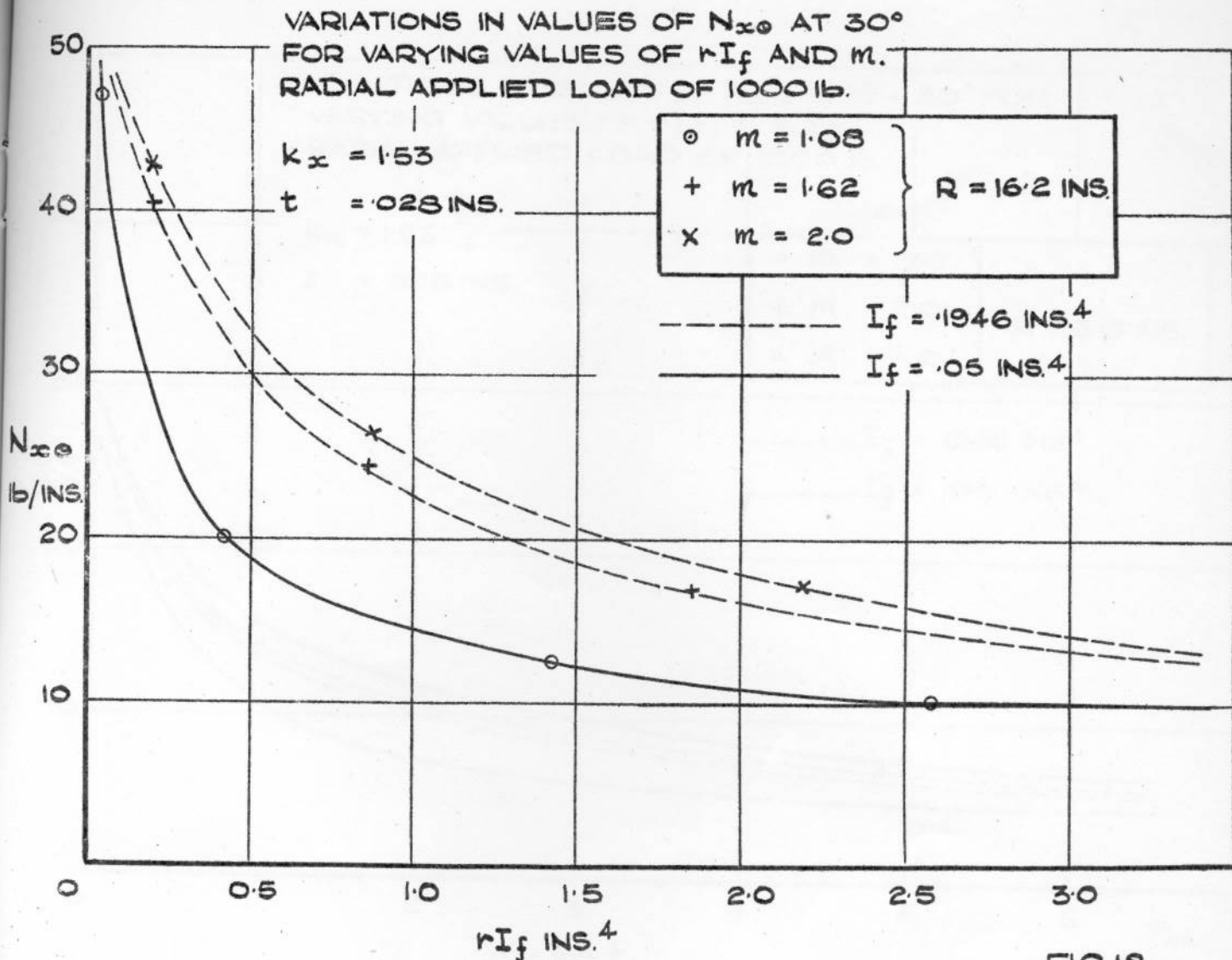


FIG.18.

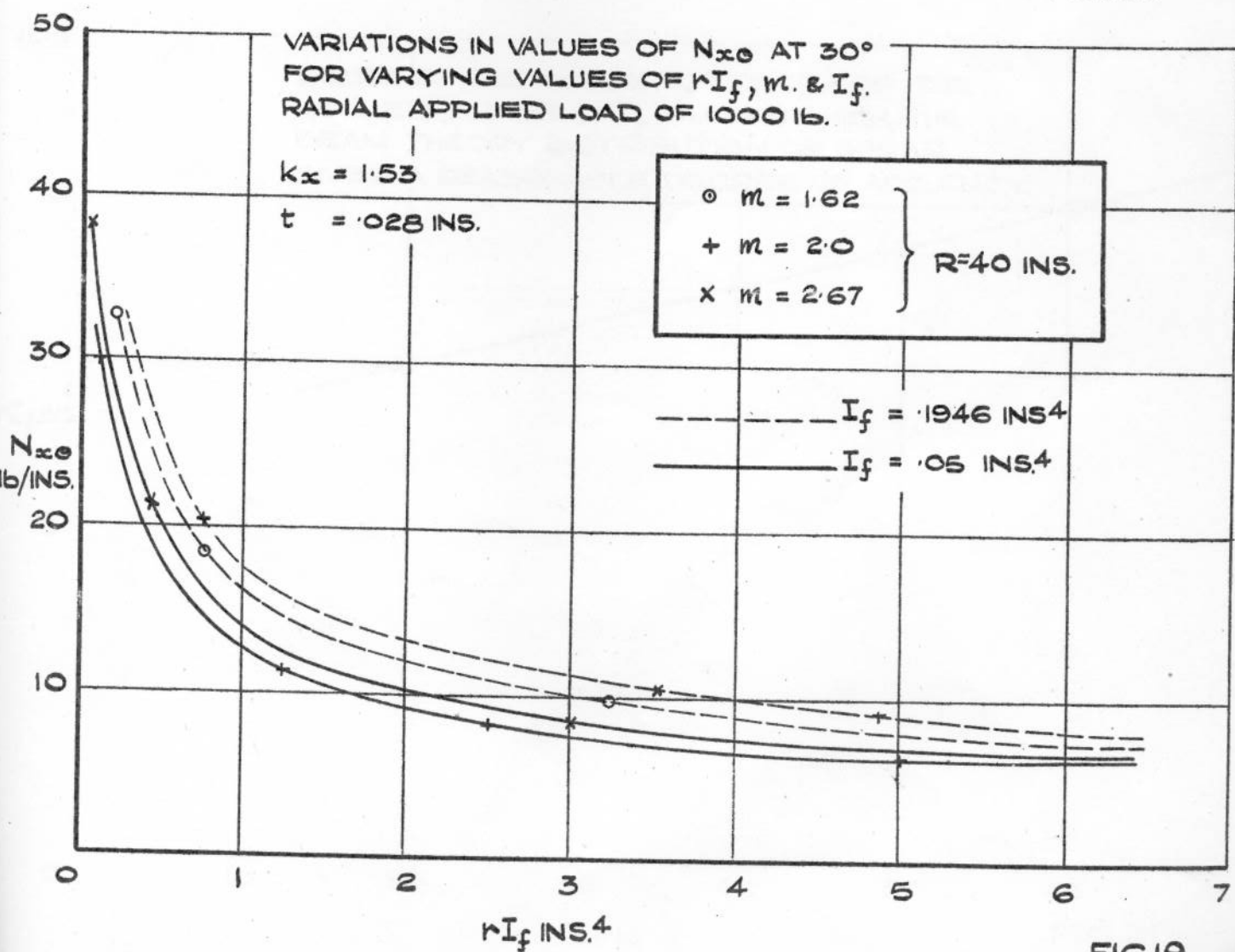


FIG.19.

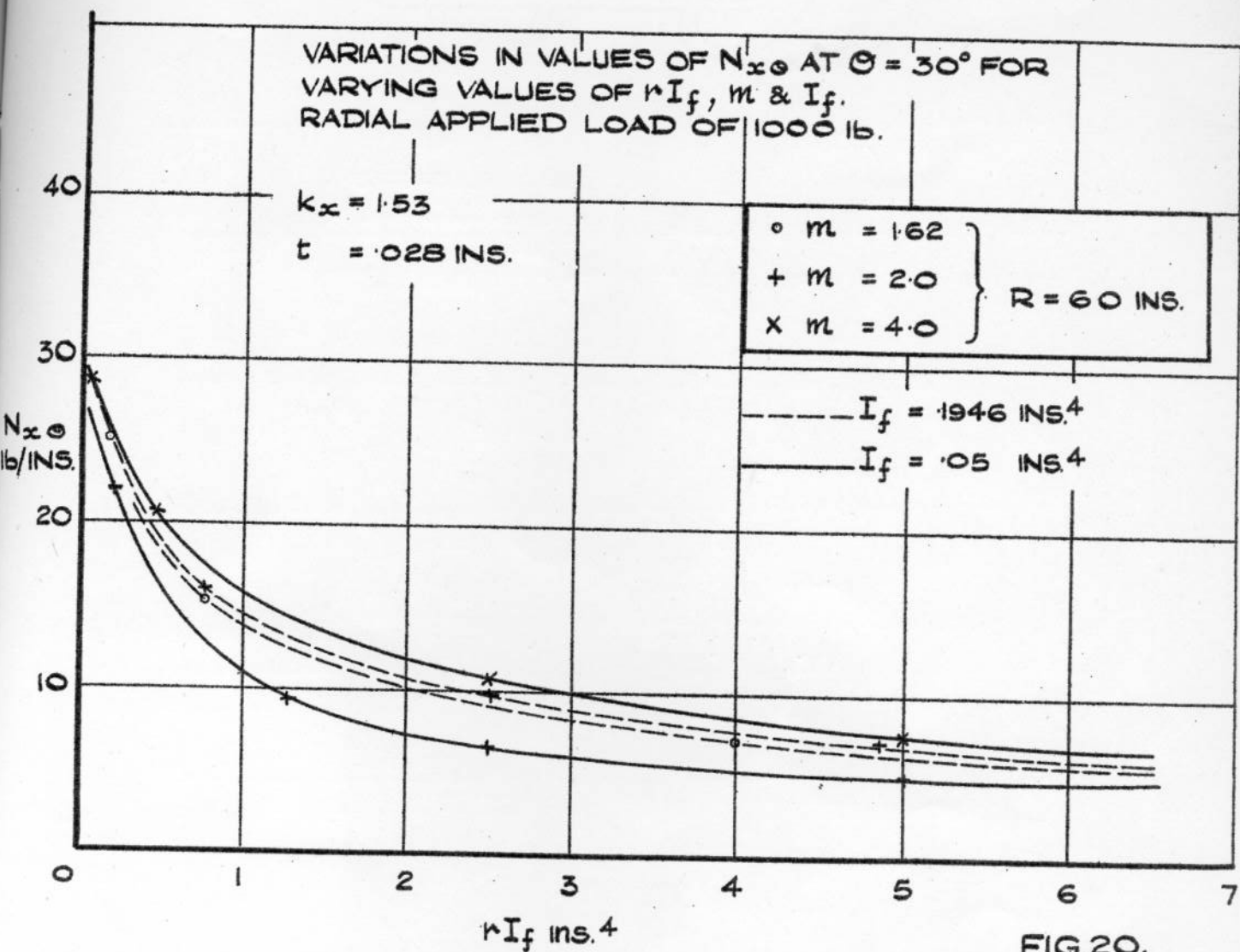


FIG.20.

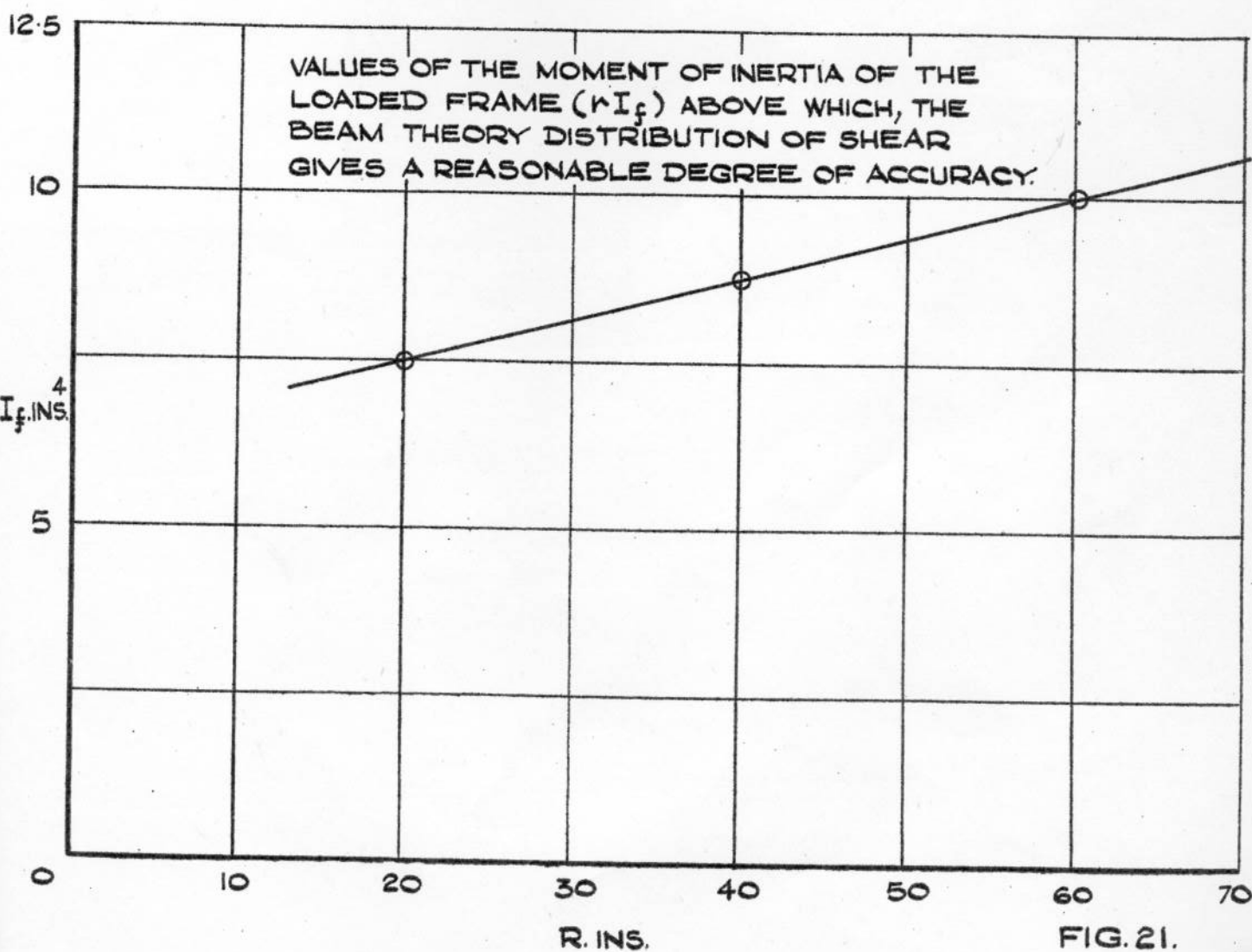


FIG.21.